Tapped-Line Coupled Transmission Lines with Applications to Interdigital and Combline Filters

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***Abstract*—Exact, general, open-wire-line equivalent circuits for tapped-line combline and interdigital arrays are derived using a combination of graph transformations and induction. A significant feature of the equivalent circuits is that they do not require commensurate length sections. The equivalent circuit for tapped-line interdigital arrays is utilized to develop design equations for tapped-line interdigital filters.**

INTRODUCTION

IN the physical realization of narrow- and moderate-bandwidth interdigital and combline filters, the conventional input and output transformer couplings are sometimes replaced with direct, tapped connections, as shown in Fig. 1(a) and (b). This method was first described in the literature by Dishal [1] for small-percentage-bandwidth interdigital filters, and recently by Cohn [2] for combline filters. Dishal's procedure is straightforward, and can be applied to both interdigital and combline filters. Analysis of the resulting designs, however, is approximate since the open-wire-line equivalent circuits for tapped-line coupled transmission lines have not been previously published. Consequently, strict computer-aided optimization is not possible and final design refinements must be done experimentally.

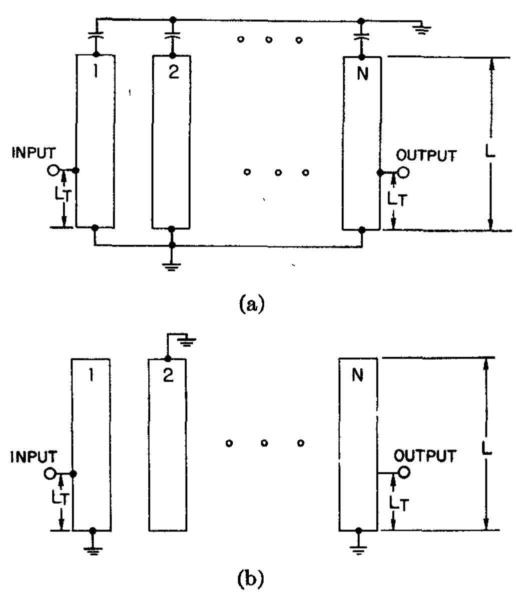

Fig. 1. Combline and interdigital filters having tapped-line input and output couplings. (a) Combline. (b) Interdigital.

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In this paper, exact, general, open-wire-line equivalent circuits for combline and interdigital arrays are developed using a combination of graph transformations [3] and induction. The graph-transformation method of analysis consists of representing a coupled-line array of commensurate-length lines by an equivalent open-wire-line circuit having no coupled lines. The open-wire-line circuit, called the network graph, is in one-to-one correspondence with the coupled-line array. Transformations are then performed on the graph to reduce the open-wire-line equivalent circuit to a simpler or more useful form.

The exact equivalent circuits obtained in this paper can be used to derive design equations for tapped-line combline and interdigital filters. For illustrative purposes, design procedures are presented for the interdigital filter, and computed results for two design examples are discussed.

EQUIVALENT-CIRCUIT DERIVATIONS

The graph-transformation process [3] is suggested in Fig. 2 for the 2-coupled-line combline case with the tap located $\frac{1}{3}$ distance from the ground connection. In Fig. 2, inductor symbols and straight bold lines represent short-circuited transmission lines and transmission lines, respectively, all of length L_T. Fig. 2(a) depicts the geometry; Fig. 2(b), the network graph; Fig. 2(c), the network graph after several graph transformations; and Fig. 2(d), the network graph in essentially final form. When the tap is positioned $2L_T$ from the short circuit, the equivalent circuit is identical to Fig. 2(d), but with L_T and $2L_T$ interchanged. If L_T were made L/N or ML/N, where M and N are integers, analogous results would be obtained. In general, for a suitable M [M a function of N]

$$\lim_{N\to\infty} | L_T - ML/N | = 0$$

for arbitrary L_T. Hence, by induction, the exact, general, open-wire-line equivalent circuit for the combline case for arbitrary lengths L_T is given in Fig. 3. Notice that the equivalent circuit contains transmission lines of lengths, L, $L - L_T$, and L_T, where the lengths L_T and L need not be commensurate. Generalization to the multiline array is obtained by inspection of the multiline graph. For example, the 3-coupled-line case is shown in Fig. 4.

Several cases can easily be examined to test the consistency of the equivalent circuit of Fig. 3 in limiting cases: 1) for $L_T = L$, Fig. 3 yields the conventional combline circuit; 2) for $y_{12} = 0$, Fig. 3 yields the correct equivalent circuit of a single tapped line; and 3) for

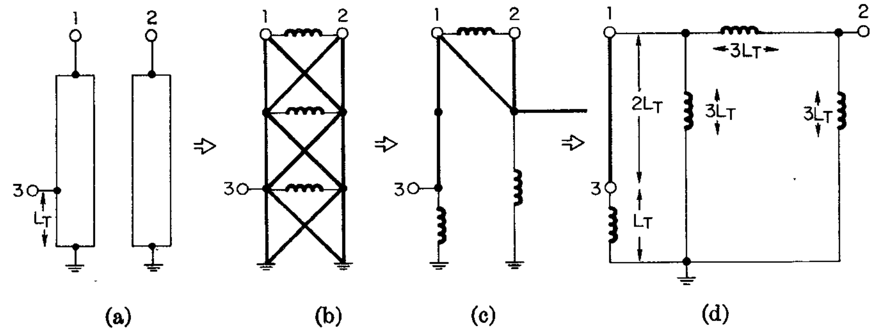

Fig. 2. Illustration of the technique used to derive the equivalent circuit for the tapped-line combline geometry.

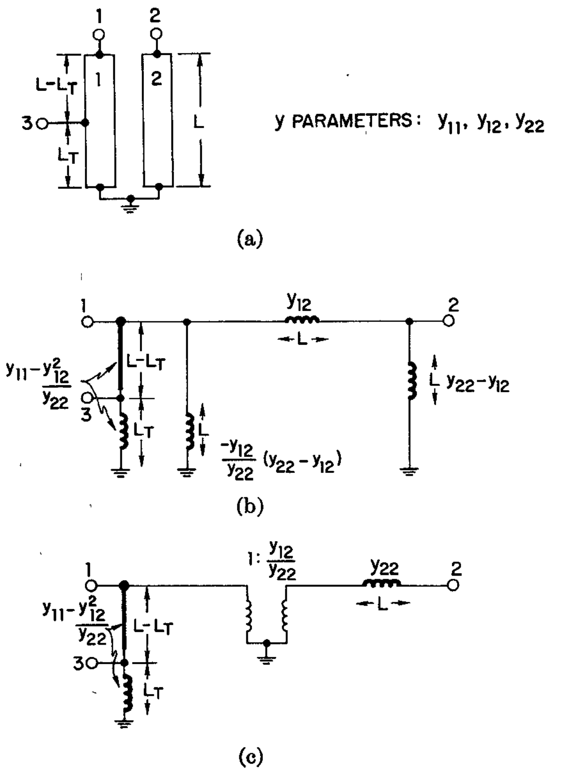

Fig. 3. Open-wire-line equivalent circuit for tapped-line combline geometry. (a) Combline line parameters. (b) Equivalent circuit with negative element. (c) Equivalent circuit with all positive elements.

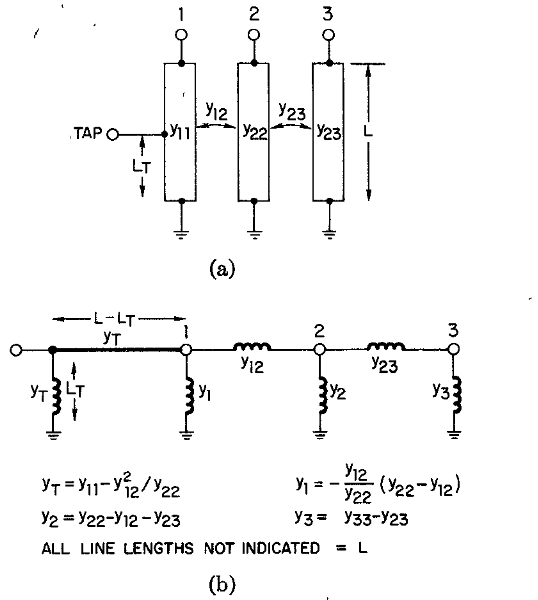

Fig. 4. Open-wire-line equivalent circuit for 3-coupled-line tapped-line combline geometry. (a) Geometry and parameters. (b) Equivalent circuit.

port 3 open, Fig. 3 yields the conventional combline circuit.

By the same procedures, the equivalent circuit for the tapped-line interdigital coupled pair was derived and is given in Fig. 5. Again generalization to the multiline array is obtained by inspection of the multiline graph. For example, the 3-coupled-line case is shown in Fig. 6.

TAPPED-LINE INTERDIGITAL-FILTER DESIGN PROCEDURES

Design equations for interdigital filters [4]–[7] and combline filters [8] may be modified to accommodate tapped-line input and output couplings. One technique is to establish an equivalence between the conventional and tapped-line coupling circuits and to replace the conventional with the tapped-line circuits. Of course, an exact equivalence is not possible over a band of frequencies, but reasonably good results can be achieved for narrow- to moderate-bandwidth filters.

Fig. 7 depicts the first three coupled lines of an interdigital filter. The first line, shown in dashes, is the input line of a conventional interdigital filter. The tapped-line input is shown connected to line 2. Fig. 8(a) gives the equivalent circuit for the conventional interdigital filter with input at port 1, while Fig. 8(b) gives an equivalent circuit for the corresponding tapped-line filter with input at port 1′. In Fig. 8(b), the exact tapped-line equivalent circuit has been utilized; however, it has been put into a more convenient form for the present derivation. The circuit enclosed in the dashed rectangle will be referred to as the "coupling circuit." Note that it is a function of three parameters: a single admittance, $Y = y_{22} - y_{23}^2/y_{33}$; the length of the line, L; and the tap length, L_T. The admittance of the tapped-line circuit viewed from the ideal transformer is

$$\frac{Y_{\text{in}}(\theta)}{G_L} = jyw + y\left\{\frac{1 + ww_2 - jyw[1 + w_2^2]}{y[1 + w_2^2] + j[w_2 - w]}\right\}$$

where

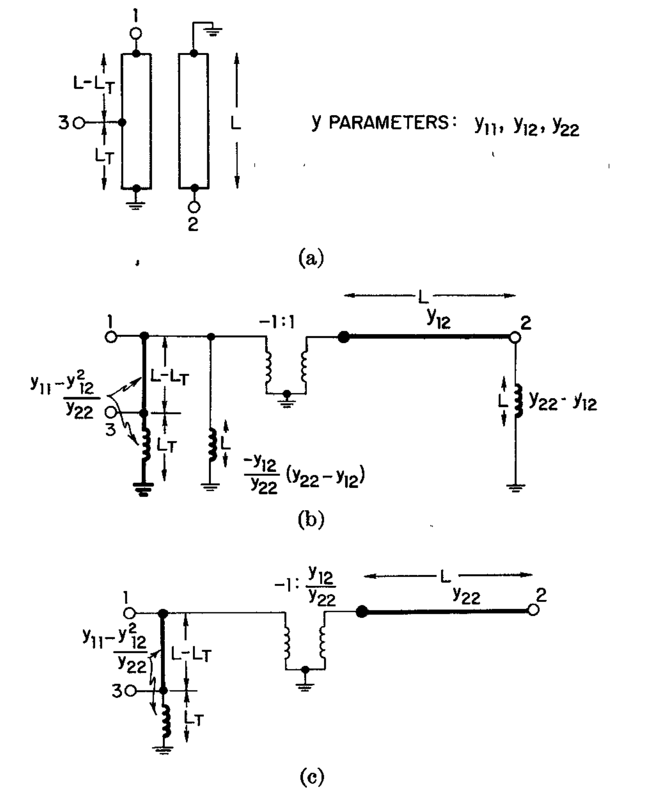

Fig. 5. Open-wire-line equivalent circuit for tapped-line interdigital geometry. (a) Interdigital line parameters. (b) Equivalent circuit with negative element. (c) Equivalent circuit with all positive elements.

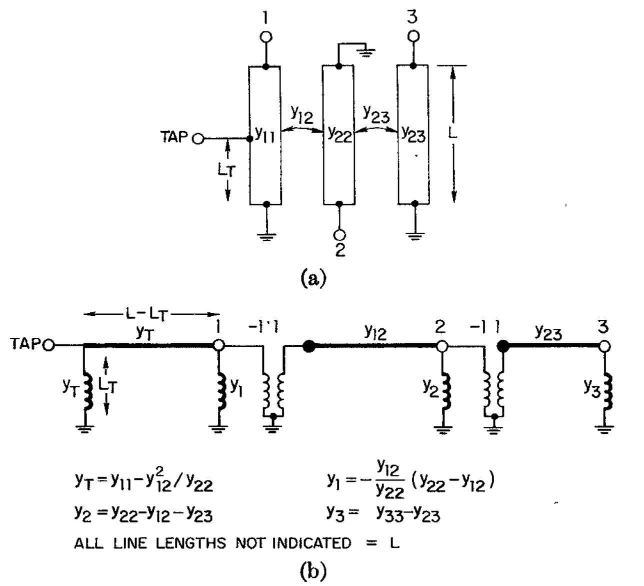

Fig. 6. Open-wire-line equivalent circuit for 3 coupled-line tapped-line interdigital geometry. (a) Geometry and parameters. (b) Equivalent circuit.

$$y = Y/G_L = \frac{1}{G_L}[y_{22} - y_{23}^2/y_{33}]$$

$$w = \cot\theta$$

$$w_2 = \cot\theta_2$$

$$\theta = \frac{\omega L}{v}$$

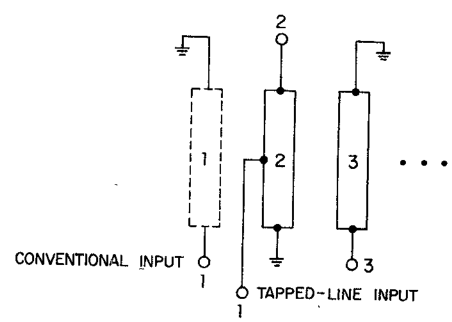

Fig. 7. First three coupled lines of an interdigital filter.

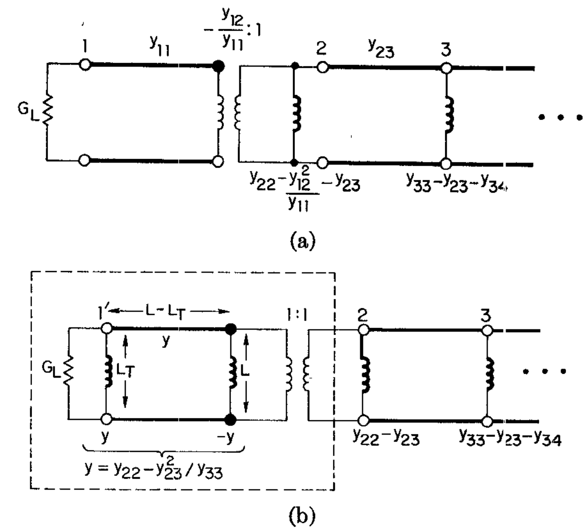

Fig. 8. Partial equivalent circuits for conventional and tapped-line interdigital filters. (a) Conventional. (b) Tapped line.

$$\theta_2 = \frac{L_T\theta}{L}.$$

If Fig. 8(a) and (b) are to be equivalent, then

$$\text{Re}\,[Y_{\text{in}}(\theta)/G_L] = (y_{12}/y_{11})^2. \tag{1}$$

Fig. 9 plots the Re $[Y_{\text{in}}(\theta)/G_L]$ versus L_T/L at $\theta = \pi/2$ for $Y = 2$ and 100. Note that the Re $[Y_{\text{in}}/G_L]$ is very insensitive to variations in Y in the previously given range. This is important because, in order to later adjust the reactance function, y_{22} will need to be modified. Fortunately, even large changes in y_{22} will not require a repositioning of the tap, as shown by the data in Fig. 9.

In addition, for Fig. 8(a) and (b) to be equivalent

$$-(y_{22}' - y_{23})\cot\theta + \text{Im}\,[Y_{\text{in}}(\theta)/G_L] = -(y_{22} - y_{12}^2/y_{11} - y_{23})\cot\theta \tag{2}$$

where in (2) y_{22} has been replaced by y_{22}' on the left side of the equation since the self-admittance of line 2 will need to be slightly modified. The Im $[Y_{\text{in}}/G_L]$ for $\theta = \pi/2$ is plotted in Fig. 10 as a function of L_T/L with Y as a parameter. Since $\cot\theta$ is zero at $\pi/2$, (2) can be satisfied only if Im $[Y_{\text{in}}(\pi/2)/G_L] = 0$. However, as shown in Fig. 10, this is not the case. Hence, (2) can be satisfied only with the addition of a lumped or distributed capacitor

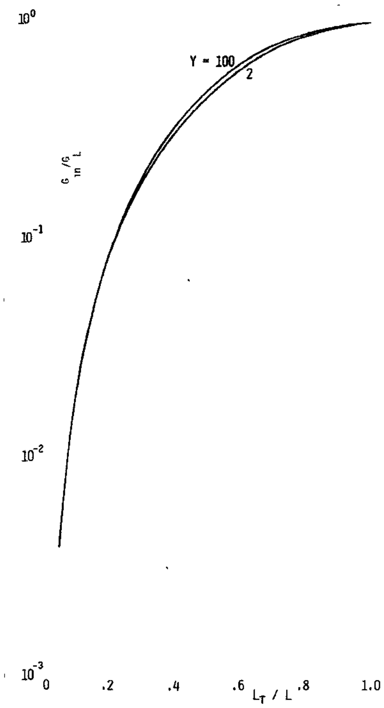

Fig. 9. G_{in}/G_L versus normalized tap length with Y as a parameter.

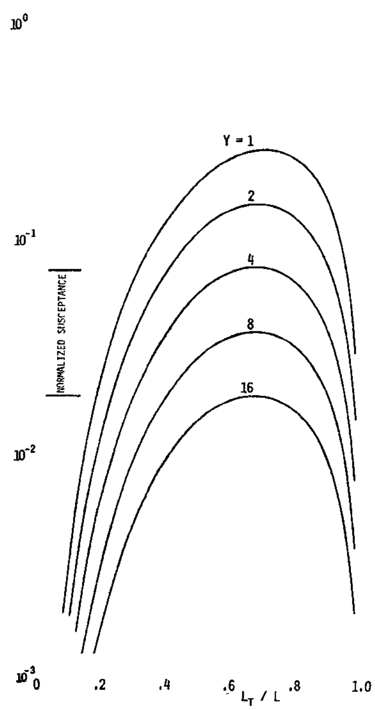

Fig. 10. Absolute value of normalized susceptance versus normalized tap length with Y as a parameter.

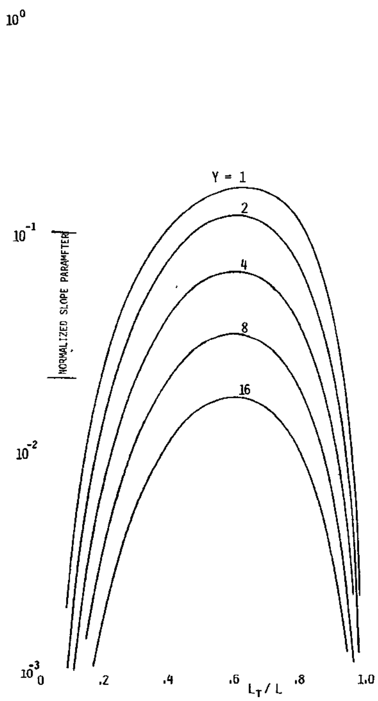

Fig. 11. Absolute value of slope parameter versus normalized tap length with Y as a parameter.

in shunt with port 2. The value of the required susceptance is read directly from Fig. 10. If a lumped capacitor is used to resonate the circuit, its value is

$$C = G_L/\omega_0 \text{ [value from Fig. 10] F.}$$

For a frequency-independent equivalence, it is necessary to equate slope parameters of the two circuits as computed from port 2. The requirement is

$$(y_{22}' - y_{23})\pi/4 + \text{"slope parameter of coupling circuit"} = \pi/4(y_{22} - y_{12}^2/y_{11} - y_{23}). \quad (3)$$

The slope parameter of the coupling circuit is negative for all values of L_T/L. Its absolute value is given in Fig. 11. In general, (3) is nonlinear. However, it may be assumed

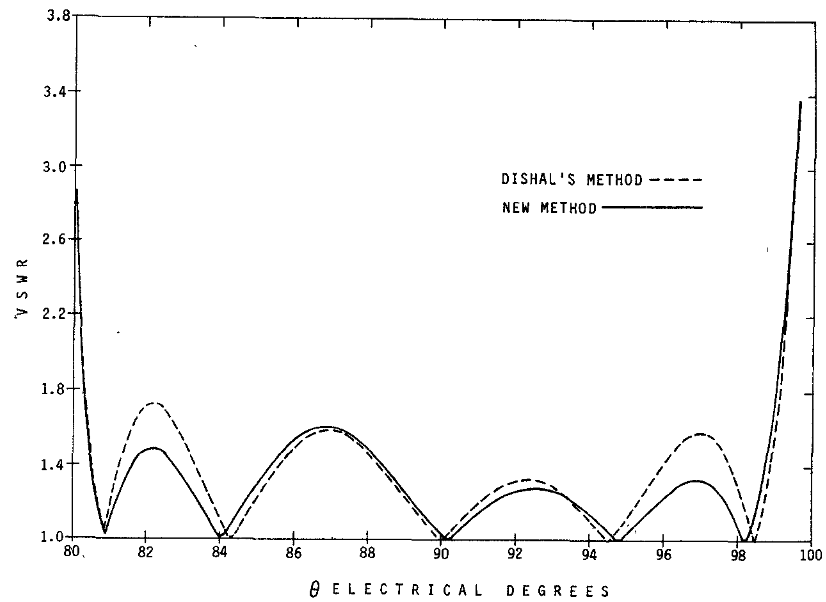

Fig. 12. Computed VSWR's; for example, 20-percent bandwidth tapped-line interdigital filters.

in most cases that the slope parameter of the coupling circuit does not vary rapidly with y_{22}'. Hence (3) may be solved by simple transposition. In some cases, a second iteration of the previously given procedure might be necessary. As noted before, even large changes in y_{22} will not require a change in the tap position.

Based on the methods just described, the several approximate design procedures for interdigital filters [4]–[7] can be modified to accommodate tapped-line couplings. As an example, the equations of Cristal [4] have been modified for the tapped-line interdigital filter and are summarized in Appendixes I and II.

EXAMPLE RESULTS

Using the design steps given in Appendixes I and II, interdigital filters of 5- and 20-percent bandwidths were designed. The filters were 5 resonators and 0.1-dB ripple. In order to assess the new design procedures, the same specifications were used to obtain designs based on Dishal's method. The several designs were analyzed using the exact equivalent circuits.

In the case of the 5-percent bandwidth filters, both responses were very similar and both were nearly within specifications. In the case of the 20-percent bandwidth filters, the new procedure gave a somewhat improved passband VSWR, although not entirely within the specifications over the entire band. The VSWR's for both 20-percent bandwidth designs are given in Fig. 12.

CONCLUSIONS

General equivalent circuits for tapped-line combline and interdigital arrays were presented. The tapped-line interdigital equivalent circuit was utilized to derive new design procedures for tapped-line interdigital filters of narrow to moderate bandwidth. The same design approach is readily extended to the combline case.

APPENDIX I

AUXILIARY EQUATIONS AND PARAMETER DEFINITIONS

N order of low-pass prototype filter;

g_i low-pass prototype filter element values, $i = 0,1,2,\cdots,N+1$;

ω_1' low-pass prototype radian frequency cutoff

w fractional bandwidth of microwave filter $w = 2(f_2 - f_1)/(f_2 + f_1)$, where f_1 and f_2 are the lower and upper band-edge frequencies of the microwave filter;

$$\theta_1 = (\pi/2)[1 - (w/2)];$$

$$\tau = \tfrac{1}{2}\tan\theta_1;$$

$$G_i = [\omega_1' g_{i-1} g_i]^{-1/2}, \qquad i = 1 \text{ and } N+1;$$

$$G_i = [g_{i-1} g_i]^{-1/2}/\omega_1', \qquad i = 2,3,\cdots,N;$$

h arbitrary positive dimensionless parameter, usually less than 1, which controls immittance level in filter interior.

$$\left.\begin{aligned} A_{11}^{(i)} &= 1 \\ A_{12}^{(i)} &= h^{1/2} G_i \\ A_{22}^{(i)} &= h[G_i^2 + \tau] \end{aligned}\right\}, \qquad i = 1 \text{ and } N+1.$$

$$\left.\begin{aligned} A_{11}^{(i)} &= h\tau \\ A_{12}^{(i)} &= hG_i \sin\theta_1 \end{aligned}\right\}, \qquad i = 2,3,\cdots,N.$$

$$B_{11} = A_{11}^{(1)}$$

$$B_{22} = A_{22}^{(1)} + A_{11}^{(2)}$$

$$B_{ii} = A_{11}^{(i-1)} + A_{11}^{(i)}, \qquad i = 3,4,\cdots,N.$$

$$B_{N+1,N+1} = A_{11}^{(N)} + A_{22}^{(N+1)}$$

$$B_{N+2,N+2} = A_{11}^{(N+1)}.$$

$$B_{12} = A_{12}^{(1)}/A_{11}^{(1)}.$$

$$B_{i,i+1} = A_{12}^{(i)}, \qquad i = 2,3,\cdots,N.$$

$$B_{N+1,N+2} = A_{12}^{(N+1)}/A_{11}^{(N+1)}.$$

APPENDIX II

TAPPED-LINE INTERDIGITAL-FILTER CAPACITANCE-MATRIX PARAMETERS NORMALIZED TO $Y_A v^{-1}$

v velocity of propagation;

Y_A terminating admittance;

L resonator length;

L_T tap position measured from resonator ground;

C_{ii} normalized self-capacitance/unit length;

$C_{i,i+1}$ normalized mutual capacitance/unit length;

C_{L_1},C_{L_N} lumped resonating capacitance;

ω_0 $= \pi[f_2 + f_1]$, center frequency of filter;

Y_1 $= B_{22} - B_{23}^2/B_{33}$, parameter for use in Figs. 9–11;

Y_N $= B_{N+1,N+1} - B_{N,N+1}^2/B_{NN}$, parameter for use in Figs. 9–11.

Normalized tap length

L_{T_i}/L equals the abscissa of Fig. 9, with ordinate equaling $[A_{12}^{(j)}/A_{11}^{(j)}]^2$ for $i = 1$, $j = 1,Y_1$, and for $i = N$, $j = N + 1,Y_N$.

Normalized Lumped Susceptance

$(\omega_0 C_i^L)/Y_A$ equals the ordinate of Fig. 10 with abscissa equaling L_{T_i}/L and parameter $Y = Y_i$, $i = 1$, and N.

Normalized Mutual Capacitance

$$C_{i,i+1} = B_{i+1,i+2}, \qquad i = 1,2,\cdots,N-1.$$

Normalized Self-Capacitance

$$C_{11} = B_{22} - B_{12}^2/B_{11} + \frac{4}{\pi} F(Y_1, L_{T_1}/L)$$

with F given in Fig. 11

$$C_{NN} = B_{N+1,N+1} - B_{N+1,N+2}^2/B_{N+2,N+2} + \frac{4}{\pi} F(Y_N, L_{T_N}/L)$$

$$C_{ii} = B_{i+1,i+1}, \qquad i = 2,3,\cdots,N-1.$$

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